SAR image processing using adaptive stack filter

María Elena Buemi *, Julio Jacobo, Marta Mejail

Universidad de Buenos Aires, Facultad de Ciencias Exactas y Naturales, Departamento de Computación, Ciudad Universitaria, Pabellón I, C1428EGA Buenos Aires, Argentina

Abstract

Stack filters are a special case of non-linear filters. They have a good performance for filtering images with different types of noise while preserving edges and details. A stack filter decomposes an input image into several binary images according to a set of thresholds. Each binary image is filtered by a Boolean function. The Boolean function that characterizes an adaptive stack filter is optimal and is computed from a pair of images consisting of an ideal noiseless image and its noisy version. In this work the behavior of adaptive stack filters on synthetic aperture radar (SAR) data is evaluated. With this aim, the equivalent number of looks for stack filtered data are calculated to assess the speckle noise reduction capability of this filter. Then a classification of simulated and real SAR images is carried out on data filtered with a stack filter trained with selected samples. The results of a maximum likelihood classification of these data are evaluated and compared with the results of classifying images previously filtered using the Lee and the Frost filters.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

The aim of this paper is to study the use of adaptive stack filters in SAR image processing. Stack filters are a special case of non-linear filters. They have a good performance for filtering images with different types of noise while preserving edges and details. Various authors have studied this type of filters and many methods have been developed (Dellamonica et al., 2007; Diaz et al., 2004; Lin et al., 1990; Lin and Kim, 1994; Paredes and Arce, 1999; Wendt et al., 1986). Many methods have been developed for stack filter design (Prasad, 2005; Shi et al., 2005). These filters decompose the input image by thresholds yielding a binary image for each threshold value. Each binary image is then filtered using a Boolean function evaluated on a sliding window. The resulting image is obtained summing up all the filtered binary images obtained. This Boolean function must be optimal according to the mean absolute error (MAE) criterion and must preserve the so-called stacking property, which will be described in Section 2. The stack filter design method used in this work is based on an algorithm proposed by Yoo et al. (1999). In this paper we study the application of this type of filter to synthetic aperture radar (SAR) images as a stage previous to a maximum likelihood classification.

SAR images are generated by a coherent illumination system and are affected by the coherent interference of the signal backscatter by the elements on the terrain (Goodman, 1976; Oliver and Quegan, 1998). This interference causes fluctuations of the detected intensity which varies from pixel to pixel. This effect is called speckle noise. Speckle noise, unlike noise in optical images, is neither Gaussian nor additive; it follows other distributions and is multiplicative. Due to all of this, it is not possible to treat these images using the classical techniques appropriate for optical image processing. Many authors have studied the problem of adapting classical image processing methods to be applied to SAR images using filter-based techniques (Lee, 1981; Lopés et al., 1993; Smith, 1996; Frost et al., 1982), with some success.

The multiplicative nature of the speckle noise leads us to model the SAR image Z as the product of two independent random images: image X, that represents the backscatter and image Y, that represents the speckle noise. The backscatter is a physical magnitude that depends on the geometry and water content of the surface being imaged, as well as on the angle of incidence, frequency and polarization of the electromagnetic radiation emitted by the radar. Different statistical distributions have been proposed in the literature. In this work we use the Gamma distribution, for the speckle, the reciprocal of Gamma distribution, for the backscatter, which results in the \( Y^0 \) distribution for the return (Frery et al., 1999; Frery et al., 1996; Mejail et al., 2003). These distributions depend on three parameters: \( \alpha \) that is a roughness parameter, \( \beta \) a scale parameter, and \( n \) the equivalent number of looks. In this work, we classify an image into different regions according to their homogeneity degree, which will be treated in Section 5. After filtering, the image data have undergone changes in their statistical distribution functions. These filtered data have a statistical distribution with skewness and kurtosis closer to those of the Gaussian law. Then, we classify the image by using the maximum likelihood method and consider the normal distribution.
with different parameters for each region. The structure of this paper is as follows: Section 2 gives an introduction to stack filters, Section 3 describes the filter design method used in this work. In Section 4 we summarise the $g^0$ distribution for SAR images. In Section 5 we show the modification undergone by the data after applying the filter, the results of classifying the filtered data and compare these results with the results of applying the Lee and the Frost filter. Finally, in Section 6 we present the conclusions.

2. Stack filters: definitions and designing

This section is dedicated to a brief synthesis of stack filter definitions and design. For more details on this subject, see Astola and Kuosmanen (1997), Coyle et al. (1989), Coyle and Lin (1988), Lin et al. (1990), Lin and Kim (1994), Wendt et al. (1986), Yoo et al. (1999). In the first place the necessary definitions are presented to explain this type of filters. The threshold operator is given by

$$T^m : \{0, 1, \ldots, M\} \rightarrow \{0, 1\}$$

$$T^m(x) = \begin{cases} 1 & \text{if } x \geq m \\ 0 & \text{if } x < m \end{cases} \quad (1)$$

$$X^m = T^m(x). \quad (2)$$

According to this definition, the value of a non-negative integer number $x \in \{0, 1, \ldots, M\}$ can be reconstructed making the summation of its thresholded values between 0 and $M$. The formula corresponding to this operation is

$$X = \sum_{m=1}^{M} X^m. \quad (3)$$

What follows shows an example of the threshold decomposition of a unidimensional signal:

$$X = [2, 1, 4, 5, 3, 2, 4, 3]$$

$$X^1 = [1, 1, 1, 1, 1, 1, 1, 1]$$

$$X^2 = [1, 0, 1, 1, 1, 1, 1, 1]$$

$$X^3 = [0, 0, 1, 1, 1, 1, 1, 1]$$

$$X^4 = [0, 0, 1, 1, 0, 0, 1, 0]$$

$$X^5 = [0, 0, 0, 0, 1, 0, 0, 0]$$

The threshold operator can be extended to bi-dimensional signals. Let $X = (x_0, \ldots, x_{n-1})$ and $Y = (y_0, \ldots, y_{n-1})$ be binary vectors of length $n$, then let us define a relation $\leq$ by

$$X \leq Y \quad \text{if and only if} \quad \forall i, x_i \leq y_i. \quad (4)$$

This relation is reflexive, anti-symmetric and transitive, generating therefore a partial ordering on the set of binary vectors of fixed length. A boolean function $f : \{0, 1\}^n \rightarrow \{0, 1\}$, where $n$ is the length of the input vectors, has the stacking property if and only if

$$\forall X, Y \in \{0, 1\}^n, X \leq Y \Rightarrow f(X) \leq f(Y). \quad (5)$$

We say that $f$ is a positive boolean function if and only if it can be written by means of an expression that contains only non-complemented input variables. That is

$$f(x_1, x_2, \ldots, x_n) = \bigvee_{i=1}^{K} \bigwedge_{P \in p_i} x_i. \quad (6)$$

where $n$ is the number of arguments of the function, $K$ is the number of terms of the expression and the $P_i$ are subsets of the interval $[1, \ldots, n]$. $\vee$ and $\wedge$ are Boolean operators AND and OR. It is possible to prove that this type of functions has the stacking property. If the function $f$ used to filter an image $X$ fulfills the stacking property, then from (4) and (5) it is deduced that, for two binary images $X^i$ and $X^j$, obtained from $X$ as the result of the application of the thresholds $T^i$ and $T^j$ respectively, the following implication is valid:

$$i \geq j \Rightarrow X^i \leq X^j \Rightarrow f(X^i) \leq f(X^j). \quad (7)$$

A stack filter is defined by the function $S_f : \{0, \ldots, M\}^n \rightarrow \{0, \ldots, M\}$, corresponding to the Positive Boolean function $f(x_1, x_2, \ldots, x_n)$ expressed in the given form by (6). The function $S_f$ can be expressed by means of

$$S_f(X) = \sum_{m=1}^{M} f(T^m(X)). \quad (8)$$

In Fig. 1 it can be observed a scheme of application of the filter to a unidimensional signal. $S_f$ represents the boolean function which filters each binary thresholded signal and whose outputs are added together to finally obtain the filtered signal.

3. Adaptive algorithms for stack filter design

In this work we applied the stack filter generated with the fast algorithm described in (Yoo et al., 1999). This algorithm arises as a result of studies on the methods proposed in (Lin et al., 1990 and Lin and Kim, 1994). To construct a stack filter following any of these methods, a training process that generates a positive boolean function that preserves the stacking property, represented by the so-called decision vector, is carried out.

In what follows, the generation and behaviour of the decision vector is explained. An image is defined as the set given by: $(s, v) : s \in S \subseteq \mathbb{Z}, v \in [0, \ldots, M], M \in \mathbb{N}$, where $s$ is the position and $v$ is the value of a pixel. Let $E$ and $R$ be two images, where $R$ is the noisy version of $E$. A window $W(s) = W_0(s), \ldots, W_{b-1}(s)$, with $W_i(s) \in [0, \ldots, M], 0 \leq j \leq b - 1$ is a subimage of $R$ of size $b$ centered at position $s$.

Let us define $V$ as a $2^b$ dimension vector, and call it the decision vector. In the training phase of the filter design process, for each position $s$ in $S$, the data in $W(s)$ are decomposed by $M$ thresholds, yielding $M$ binary windows $W^i(s), i = 1, \ldots, M$, where $W^i(s) = [W^i_0(s), \ldots, W^i_{b-1}(s)]$, and $W^i_j(s) = T^i(W(s)), 1 \leq i \leq M$.

On the other hand, $E(s)$, the value of ideal image $E$ at position $s$, is also decomposed by $M$ thresholds, yielding $M$ binary values $E^i(s) = T^i(E(s)), 1 \leq i \leq M$.

Vector $V$ will suffer $M$ updates for each pixel in the image. For each pixel position $s \in S$ and for each threshold $M$, only one position $k$ on vector $V$ will be updated. This position $k$ will result from considering the contents of binary window $W^i(s)$ as a binary number and comparing it with $E^i(s)$.
ber, that is \( k = \sum_{i=0}^{q-1} W_i^j(s)2^i \). The update will consist of an increment or decrement by one depending on the corresponding binary value \( E(s) \). So, the \( p+1 \) update will be given by

\[
V^{p+1}(k) = \begin{cases} 
V^p(k) + 1 & \text{if } E(s) = 1 \\
V^p(k) - 1 & \text{if } E(s) = 0 
\end{cases}
\]

where \( k = \sum_{i=0}^{q-1} W_i^j(s)2^i \), leaving all the other positions of vector \( V \) unchanged.

After this training phase, a boolean function could be obtained by assigning the value 1 to the positions of the decision vector \( V \) that hold positive values and by assigning 0 to the other positions. But if we did this, we would not obtain a boolean function that satisfies the stacking property, i.e., a positive boolean function. So, a phase for checking and enforcing this property on the decision vector \( V \) is necessary.

There are various methods to do this. Among them, we chose the parallel method proposed in (Yoo et al., 1999). As an example of this method, Fig. 2 shows the decision vector corresponding to a sliding window of size 3, together with the successive stages in which positions at a Hamming distance of one are compared. Whenever a violation of the stacking property is detected, the values of the offending positions are adjusted according to the following rule

\[
d_i^{p+1} = \left[ \frac{d_i^p + d_j^p}{2} \right], \quad j = i-1
\]

where \( i \) is the stage number, positions \( i \) and \( j \) are such that \( i - j = 2^p \) and \( d_j^p > d_i^p \). A thorough proof of correctness of this approach can be found in the mentioned article.

After a series of training phases followed by the corresponding stages for checking and enforcing the stacking property, the decision vector \( V \) converges to its final value. Then, a positive boolean function is obtained by thresholding the decision vector \( V \) with a threshold equal to zero.

### 4. SAR images: the multiplicative model

In this section we introduce the statistical laws commonly used under the multiplicative model for Synthetic Aperture Radar (SAR) images. The multiplicative model considers the image returned by the SAR, named \( Z \), as a product of two independent random variables, one corresponding to the backscatter \( X \) and the other one corresponding to the speckle noise \( Y \), so

\[
Z = XY
\]

where we suppose independence among the random variables corresponding to each image pixel. We can write formula (11) for each pixel \((i,j)\) of an image of size \( M \times N \) as

\[
Z_{ij} = X_{ij}Y_{ij}, \quad 0 \leq i \leq M-1, \quad 0 \leq j \leq N-1,
\]

and, the type of target each pixel belongs to (forest, pasture, crops, city) determines the most appropriate distribution for each of the backscatter random variables \( X_{ij} \).

The speckle noise comes from the coherent addition of individual returns produced by elements present in each resolution cell. So, for example, in an image corresponding to a scene of land covered by vegetation, the returns from the elements of the plants and the ground are added taking into account the phase, yielding as a result a complex number. In an amplitude SAR image, the gray level of each pixel is the module of this complex number. In an intensity SAR image, the gray level of each pixel is the square of this magnitude.

For every pixel, the model for the speckle noise is the \( \Gamma(n,n) \) distribution, where \( n \) is the equivalent number of looks. Then, within this model the density function for the speckle noise \( Y \) is given by

\[
f_Y(y) = \frac{n!}{\Gamma(n)} y^{n-1} e^{-ny}, \quad y \geq 0.
\]

In SAR images the minimum for \( n \) is 1. This value corresponds to images generated without making the average of several looks. Images generated in this manner are noisier than those generated with more number of looks, but they have better azimuth resolution and, therefore, potentially more information. We can suppose that the parameter \( n \) is known or that it can be estimated at an initial stage of the image analysis. Therefore, although in theory it would have to be an integer number, in practice it is necessary to consider it as a real number for the case in which it is estimated from the data.

The moments of the speckle distribution are given by

\[
E[Y^n] = \frac{1}{n^r} \Gamma(n+r)
\]

where \( r \) is the moment order and \( n \geq 1 \) is the number of looks.

There are several models for the backscatter, that is, different statistical distributions exist for the random variables \( X_{ij} \). From the results presented in (Frery et al., 1996) it is possible to consider the Generalized Inverse Gaussian distribution as a general model for the backscatter. This distribution is very general and allows us to describe many different targets, but from an analytical and numerical point of view the estimation of its parameters is very complex and unstable. This distribution has various particular cases, one of which: the Inverse Gamma distribution, is of special interest to this work. This distribution is proposed as a universal model for SAR data and it leads to the density function for the return. The Inverse Gamma distribution, called \( \Gamma^{-1} \), is characterised by the density function given by

\[
f_X(x) = \frac{2^a}{\gamma^{2a} x^{a-1}} \exp \left( -\frac{x}{\gamma} \right), \quad x > 0, \quad -\alpha > 0, \quad \gamma > 0.
\]

and its moments are expressed as

\[
E[X^r] = \frac{\Gamma(a-r-1)}{\Gamma(a-1)}, \quad r > 0.
\]

where \( z < 0 \) and \( |z| > a \). The distribution corresponding to the return \( Z \) is fixed by the distribution of the backscatter \( X \) and the distribution of the speckle \( Y \). Given that \( Z = XY \) and that these random variables are independent, \( f_Z(z) \) can be calculated as
where \( f_{Z|Y}(y) \) is the density for the return \( Z \) considering \( Y = y \) constant and \( f_Y \) the density function of the speckle \( Y \). For the random variable corresponding to the return (intensity format) we have that 
\[ Z \sim \theta^0(x; y, n), \]
and the density function is given by
\[ f_Z(z) = \frac{n!}{(y + n)!} \left( \frac{y}{z} \right)^{z-1} n^{-n} \left( \frac{y}{z} \right)^{z-n}, \quad x > 0. \]
with \( x < 0, y > 0 \) and \( n \gg 1 \). Given the independence between the backscatter \( X \) and the speckle \( Y \), the moments of the return \( Z \) are the product of the moments of \( X \) and the moments of \( Y \) (Eqs. (16) and (14)) yielding
\[ E[Z^r] = \left( \frac{y}{2n} \right)^{r} \frac{\Gamma(-r)}{\Gamma(-x)} \frac{\Gamma(n+r)}{\Gamma(n+x)} (n), \]
recalling that these moments are finite for \( -x > r \). A statistical tool for characterizing the signal to noise ratio is the variation coefficient, defined as the ratio between the standard deviation and the mean value: 
\[ CV = \frac{\sigma}{\mu}. \]
The variation coefficient is given by
\[ CV = \frac{\sigma}{\mu} = \frac{\sqrt{\frac{2(\alpha + n + 1)}{\alpha}}}{\alpha}, \quad \alpha < -(n + 1), \]
where
\[ \sigma^2 = \frac{\gamma^2}{4}(\alpha + n + 1), \quad \mu = \frac{-\gamma^2}{2}. \]
In the case of homogeneous areas and for intensity data, the equivalent number of looks (ENL) is calculated as
\[ ENL = \left( \frac{1}{\alpha^2} \right)^{1/2} = \frac{\mu^2}{\sigma^2}. \]

This quantity is a usual measure of the speckle reduction obtained by different SAR image processing algorithms.

The distribution used in this paper was proposed in (Frery et al., 1996) as a model for extremely heterogeneous data, but its utility for description of a great variety of natural and artificial targets was verified, which resulted in its being proposed as a universal model for SAR data.

### 5. Results

This section is dedicated to show the results of applying a stack filter to simulated and real SAR images. The simulated images are generated in such a way that their data have different degrees of homogeneity. We consider different values of the \( \alpha \) and the \( \gamma \) parameters. The \( \alpha \) parameter corresponds to image roughness (or heterogeneity). It adopts negative values, varying from \(-\infty\) to 0. If \( \alpha \) is near 0, then the image data are extremely heterogeneous (for example: urban areas), and if \( \alpha \) is far from the origin then the data correspond to a homogeneous region (for example: pasture areas), the values for forests and crops lay in-between. In order to evaluate the behaviour of the filters we carried out an equivalent number of looks (ENL) estimation on real SAR images and then a maximum likelihood classification on real and simulated SAR images.

#### 5.1. Statistical analysis

An important task in statistical analysis is the characterization of the mean value and the variability of a data set. To this end, the behaviour of some statistics for filtered and non-filtered images, is compared. A set of images of \( \theta^0 \) distributed data were generated using the values of \( \alpha \) and \( \gamma \). The \( \alpha \) parameter varied be-
tween $-1.5$ and $-8.5$, and the $\gamma$ parameter was adjusted so as to keep the mean value equal to one.

In order to design the stack filter, an image formed with the mean values of each region, as described in Section 3, is used. Examples of these images are shown in Fig. 3. This figure shows the results of applying an adaptive stack filter with a $3 \times 3$ window to images with different $x$ values. Fig. 3a and c correspond to the original speckled images and Fig. 3b and d correspond to the filtered images.

It is observed that the value of $CV$ for the filtered images is lower than the value of $CV$ for the non-filtered images. This indicates that the effect of filtering is a decrease in the speckle noise. It is also observed that the value of $CV$ is lower when the $x$ parameter is lower. For instance: for $x = -1.5$ the values of $CV$ are 55.2 and 26.9 for the non-filtered and filtered images, respectively; for $x = -8.5$ the values of $CV$ are 33.5 and 11.6 for the non-filtered and filtered images, respectively.

As $\gamma$ is a scale parameter, the emphasis of this study is put only on the influence of the $x$ parameter. The distribution of the data is modified when they are filtered. As a measure of asymmetry and peakedness we take into account the values obtained for skewness and kurtosis, respectively. The values obtained for these statistics indicate that filtered data are more smoothed than non-filtered data. This improves the results obtained by maximum likelihood.

Fig. 5. (a) pasture data histogram, (b) forest data histogram of the original image shown in Fig. 4.

Fig. 6. (a) pasture data histogram, (b) forest data histogram from the filtered data.

Fig. 7. Training regions (horizontal lines fill) and test regions (vertical lines fill).
classification. Note that for a normal distribution the skewness is zero and the kurtosis is 3.

It can also be seen that for high values of $\alpha$ (heterogeneous data) the filtered data are less Gaussian than for lower values of $\alpha$. This can be seen in Table 1, for $\alpha = -1.5$ and $\alpha = -8.5$.

5.2. Stack filtering speckle noise reduction on real SAR images

In order to assess the speckle noise reduction, 1-look amplitude C-band VV SAR data acquired by the DLR’s E-SAR sensor over the zone of Gilching, Germany (see Fig. 4), were used. In this image, we can observe several parcels with different types of crops, large areas covered by forests and buildings, and a grass area surrounding the air strip in the center of the image. The regions indicated on Fig. 4 correspond one to pasture and the other to forest. The former is an example of very homogeneous data, and the latter is a case of heterogeneous data. On Fig. 5 the histograms of the two aforementioned areas are shown. Here we can see the difference in variance for the two cases. The stack filter was trained with these two types of ground cover. Fig. 6 shows the histograms of the same areas after processing. The equivalent number of looks before and after filtering, for both areas can be seen on Table 2. Here we can appreciate that the reduction in variance due to filtering is consistent with the increase in the ENL obtained, for both cases. As filtered data are less noisy than the unfiltered data, the results of classifications performed on them are expected to be better.

5.3. Maximum likelihood classification of real and simulated images

For this study, one hundred images, of size 128 x 128, were generated. These images had two regions: one simulating homogeneous data (for example, pasture) and the other simulating heterogeneous data (for example, city). The data in the first region are well described by $\mathcal{G}(-8.5, 1, 1)$ distribution, and the data in the second region by a $\mathcal{G}(-1.5, 1, 1)$ distribution. As an example, Fig. 8a shows these two regions. Fig. 8b shows the result of a maximum likelihood classification on the image of Fig. 8a, c and d are the filtered and classified image, respectively. From these images, the influence of stack filtering on maximum likelihood classification performance can be assessed. Table 3 shows the average confusion matrix where: $R_i/R_j$ means the percentage of pixels that belong to region $R_j$ but were classified into region $R_i$. From these
values it can be seen that the classification performance was better for filtered images than for non-filtered images.

In order to test the proposed methodology on real SAR data, a 256 × 256 1-look subimage was extracted. The adaptive algorithm was applied to an ideal image which consists of three regions with uniform values. Each of these values is computed as the mean value of the corresponding region in the original SAR image, see Fig. 9a and b. Taking the original SAR image and the ideal image as inputs of the algorithm, the adaptive filter was generated. Then, this filter was applied ninety five times. At each iteration, the output image of the previous iteration was taken as the input image for the present one. Fig. 9c and d shows the resulting images after 1 and 95 applications of the stack filter, respectively. Fig. 10 shows the original image classified, and the classified images corresponding to 1, 40 and 95 iterations. Table 4 shows the confusion matrices for the stack filter-based methods (1, 40 and 95 iterations), and for the Frost and the Lee filter-based methods. They are based on the training regions and the test regions depicted in Fig. 7. These two filters were selected as a means of comparison of the performance of the proposed methodology. Data must be read as follows: $R_i / R_j$ means the percentage of pixels that belong to region $R_j$ but were classified into region $R_i$. From these values it can be seen that the classification performance was better for filtered images than for non-filtered images. It can also be seen that classification performance improves as the number of iterations is increased (see columns corresponding to $R_i / R_i, i = 1, 2, 3$).

6. Conclusions

In this work, the effect of adaptive stack filtering of SAR images over speckle noise reduction and classification accuracy was assessed. Synthetic and real images were used and the performance of repeatedly applying an adaptive stack filter was contrasted with the performance of Lee and Frost filtering. The preliminary results obtained show that, for the real images used for this study, consisting mainly of crop patches, the adaptive stack filter exhibits a good performance.
References


